
Physics and Function of Magnetic Resonance Imaging: Analysis of Wavelength, K-Space, Amplitude, Basic Pulse Sequence and Its Diffusion Physics

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ABSTRACT

Magnetic resonance imaging is becoming necessitate part of the tissue analysis in the medical imaging. It is the most efficient and convenient way to monitor the cellular structure of the tissues. It is important to understand MRI physics, therefore, the magnetic wave generation is needed to be addressed. The main parameters in the physics of the MR images, i.e. amplitude, frequency, phase, diffusion physics are illustrated and how this parameters are implement in the images are described. The mathematical part of the MRI is also have significant effect in the image generation. The Fourier transform is the main basis of the mathematical part related to the image generation. This paper presents a well-established knowledge that forms the basis for understanding the physics of the MR images.

Keywords: Magnetization; K- Space; Magnetic Resonance Images; Frequency; Amplitude; phase;

INTRODUCTION

One of the most efficient ways of analyzing the soft tissue in nowadays medical procedure is magnetic resonance imaging (MRI) which provides tissue contrast in order to check its morphology [1].

In the MR images, the images are generated with the help of electromagnetic waves [1]. The magnetic wave is generated using the proton oscillation, where the images are generated using contrast reflecting proton density [2], T1 and T2 relaxation times [3] and diffusion [4-6] and many other biological parameters. One of the important merit of the MR images, is that the whole procedure is noninvasive, which means that the images are generated without any harmful effects to the tissue [6-7]. The overall mathematical studies and original works of the Fourier [2], Lauterbur [8] and Mansfield [9-10] led to such an advanced and useful technology. The principle of the MRI is more complicated in comparison to the other medical imaging like X-Ray images, computed tomography (CT) etc. [9-10]. In order to understand the complex physics of the MR image formation, there should be some knowledge related to the electromagnetic waves and its generation as well as the mathematical part which is about Fourier transformation [11]. The aim of this paper is to understand the fundamental physics of the MRI. The main parameters affecting the MRI like the wave length, K-space, amplitude, basic Pulse sequence, and its diffusion physics will be described. The method of the image generation and the parameters affecting the feature of the images will also analyzed. Then this discussion is followed by the parameters and the analysis of the diffusion related phenomena. It also should mention that in order to have a matched unit similar to the biological

dimensions, the unit of length and time are expressed in micrometers and milliseconds, respectively. The diffusion coefficient of water is $D = 4 \mu\text{m}^2/\text{ms}$ at 37°C . The Larmor frequency gradient (rad/s), γ , describing the effects of gyromagnetic to be included in gravity coefficient, is defined in $(\mu\text{mms})^{-1}$.

HOW MR IMAGES ARE GENERATED?

For the formation of the medical magnetic resonance (MR) imaging, a hydrogen atom (1H) is needed. In the nucleus of the hydrogen atom, a single electron and a single proton exists. The single electron with negative charge is orbiting around the nucleus and the proton exists in the nucleus so the overall hydrogen atom is electrically neutral (Figure 1). For the formation of the MRI, the protons are main components and play an important role in this field.

It also mentioned that like almost all the elementary particles, the proton has a charge and spin. The proton rotates around its axis. When a particle rotates, two important physical properties should be considered i.e. rotation mass (m) and the angular momentum. The rotation of the proton around its axis means that it has a rotation top and the proton angular momentum tries to hold the spatial rotation around its axis. This procedure is shown in Figure 2.a. Since proton has an electrical charge and rotation around its axis, it produces the magnetic field. The rotational effect will cause an additional moment, called magnetic moment acting like a magnet. The schematic of the generated magnet is shown in Figure 2.b. Both electrical and magnetic fields generate electromagnetic waves. Therefore, the present of the moving proton will cause an electromagnetic waves which travels in the space. When the generated electromagnetic waves are received by a coil, the electrical current is generated in the coil resulted from an induced voltage.

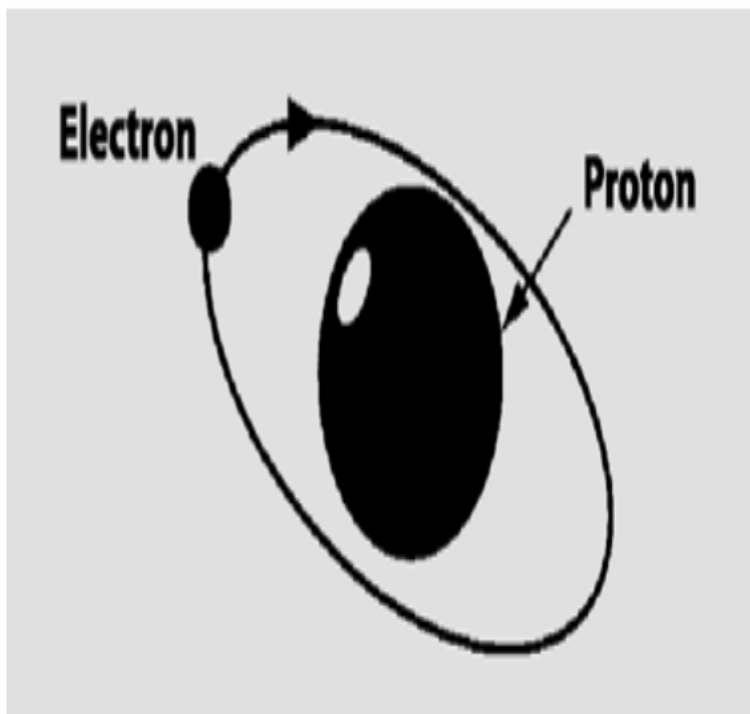


Fig. 1. Atom of the hydrogen. Hydrogen atom consists of an orbiting electron and a proton .which stays in the nucleus

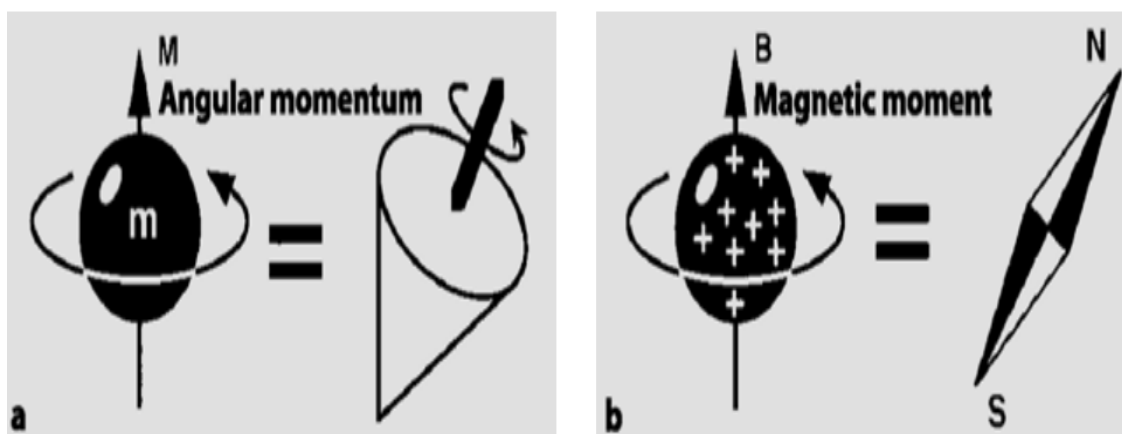


Fig. 2. The effect of the proton rotation. a) The proton rotates around its axis and its angular momentum tries to retain this rotation. b) The rotation of the proton will cause a magnetic field which results in a magnetic field generation

Analysis of the Amplitude, Phase, Frequency and K-Space

As previously mentioned, when the proton with its positive electrical charge rotates, it will generate the electromagnetic waves. When the travelling electromagnetic waves are received by a coil, it will cause an induced voltage in it. Therefore, for the purpose of MRI generation, pair of coils is used. The schematic of the coils are given in the Figure 3.a. The coils are joined to sensitive amplifiers so that the received signal will enhance and tuned to the Larmor frequency.

The generated electromagnetic waves have two components i.e. transverse and longitudinal components. The longitudinal component is in parallel to the B_0 field and the transverse component is in the opposite of the longitudinal component. The rotation of the proton which make the time varying magnetic field will induce a small amount of signal in the coils, therefore amplifiers are used to increase the amplitude of the signals. The axis of the coils is so important. The coil should align in the way that the electromagnetic wave can reach. When the electromagnetic wave reaches the perfectly aligned coils, the transversal wave should couple with the coils. The rotation of the proton will cause a variable magnetic field which led to the variable electromagnetic transversal waves and result in a desired signal.

The amplitude of the signal is related to the variation of the magnetization induced at the coils. When the two coils are used, like the way is shown in the Figure 3.a., it is possible to conclude the direction of the rotation and the angle by which the transversal wave propagates. As can be observed in Figure 3.a, the phase of the induced signals, is rotate ca. 90° . The signals are depicted with dashed and solid lines. Also the angle which declares the rotation of the transverse magnetization parameter should also be obtained. This angle of the rotating for transverse magnetization is a function of time and can be obtained by the ratio of the signals. The alignment of the bulk magnetization is in the parallel to the B_0 field. This alignment is along the Z axis. Therefore, when there is no disturbance, there will be no component of the magnetization along the transversal plane and this will in turn result in no signal generation. When the bulk magnetization does not have any component in the other planes, it is mentioned that the bulk magnetization is in the equilibrium situation. Hence, the bulk magnetization

should be shifted in a way that it would have a component align in the transversal direction. For this purpose, the spins of the protons should be tipped away. The spins are altered using a variable magnetic field (Figure 3b). The variable magnetic field, called B1 field should have the similar Larmor frequency. It should be mentioned that in the Figure 3b, only one coil is depicted for the sake of simplicity and for the signal excitation, there should be two coils oriented by 90° apart from each other, like the alignment illustrated in Figure 3a. The overall configuration will result in a more efficient RF energy production. The term RF is used here again since the Larmor frequencies are usually in the range of MHz, which is similar to the frequency of the radiofrequency in the spectrum of the electromagnetic waves. When the magnetic field varies, the protein spins absorb energy and divaricate from their original orientation, and hence, they will cause a transversal electromagnetic wave component which alters versus time. The change angle depends on the amplitude and duration of the radiofrequency. Therefore, the precise angle must be chosen so that the magnetization will be in the direction of the Z axis. It also worth noting that, in order to have the longitudinal axis of the magnetization, the frequency of the B1 field should be the same of the Larmor frequency.

Another important parameter is transverse relaxation time which determines the frequencies that can initiate the excitation. The shorter the excitation time, the greater the spread. Therefore the range of the tissues frequency must take into the consideration. The ranges of the frequencies in the tissues are usually between 1-100 Hz.

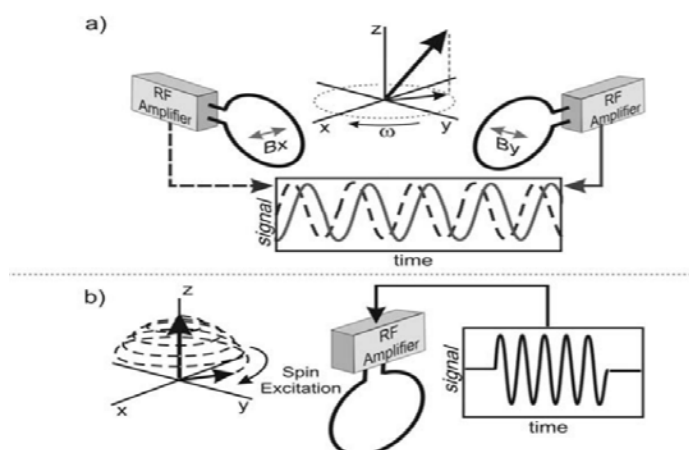


Fig. 3. Induced signals via Rotating magnetization. The signal is depended on the frequency ω , coils and the amplifiers. (a) The coils are aligned at 90° respect to the rotating magnetization. (b) Generation of the transversal magnetization. By applying a variable magnetic field, the magnetization will deviate from the Z axis.

MATERIAL AND METHOD

Formulation of the MR Images, the effect of K-Space, Phase, Amplitude and Frequency

The MR images formation is dedicated to the achievements of the Jean Baptiste Joseph Fourier (1768–1830) who is well-known for his excellent mathematical formulation, called Fourier series. The Fourier transform is also part of this mathematical formulation which can apply to many branch of the physics. The mentioned Fourier transform also plays an important role in MRI. In order to comprehend the effect of Fourier transform in the MR images, the fundamental of image generation will be discussed briefly. As an example, suppose that it is desired to draw a portrait of Fourier, depicted in Figure 4a. The picture is drawn via a

computer program. The usual computer sketch is draw using the common drawing tools like pencils, paintbrushes. With the aid of such graphic tools, certain lines at the specific locations are drawn. The overall configuration of the portrait is made of overlaying these lines. In order to form an image using the electromagnetic waves, a new tools showing in Figure 4b must be used. In the MR images, the strip pattern is formed by the oscillation received by electromagnetic waves. The positive or negative oscillation will appear as the gradient of white and black colors across the image. The color of these patterns can be adjusted by their density, phase, angle and amplitude.

The overall process of the MR image formation is shown in Figure 5, where images are formed by overlaying the various stripes. In the first step of the MR image generation, the pattern of the stripe should be selected. This selection is related to the stripe density, specific angle, phase and the related amplitude. The result of the selected pattern is shown in a red box at the right bottom corner of the Figure 5a. Next, the selected pattern is then applied to the whole graphical area. As it can be seen in Figure 5a, the wavelength of the stripe pattern, shown with the blue arrow, is obvious from the graphical area. As it can be seen the wave length of the stripe is 1 which means that the stripe density is 1/1. Hence, the more stripes on the graphical area interpret as higher density. This concept is known as "spatial frequency" of the stripe pattern, which is related to the number of the gradients per unit length. The blow arrow determines the orientation of the stripe in terms of its angle. Also the stripe pattern contrast variation in brightness relates to the stripe amplitude. Two sets of similar stripe patterns, in terms of density, amplitude and orientation, are put beside each other to generate the MR image. The difference exists between two pattern is related to the a quarter shift of one stripe pattern relative to the other one. Also phase can be determined with the measurement of the relative size of these two patterns. Therefore, for formation of the MR images, the amplitude, density, orientation and phase of the stirpes as well as the pattern of the drawing tools should be determined.

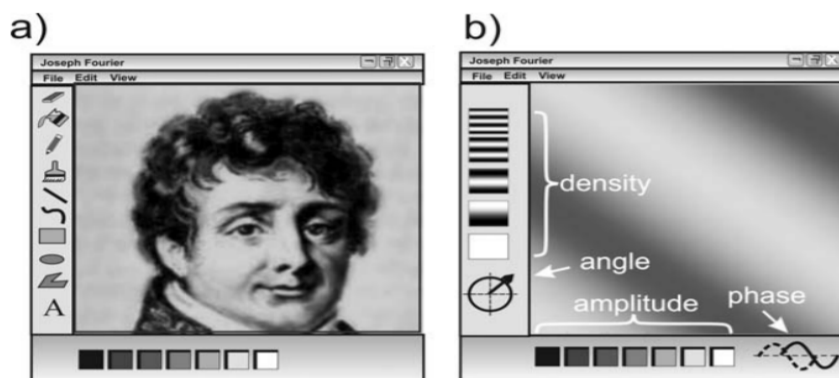


Fig. 4. The common tools for drawing an image (a) drawing the portrait of Fourier (b) graphical tools used to form MR images.

The next step in image formation is to add another stripe pattern to the first one. Again the amplitude, angle, frequency and the phase of the second stripe are very important and should be selected, precisely. As can be seen in the Figure 5b, the second pattern stripe, shown in the right bottom part in the red box, is added to the graphical area. It can be seen that the second stripe pattern has lower amplitude, phase, angle and spatial frequency. In fact the angle, is in its horizontal format. The outcome of superimposing these two patterns results in a patchwork

with variation in density. As an example, if 500 stripe patterns are put beside each other in the graphical area, the result would be similar to what is shown in Figure 5c. It should be mentioned that the frequency, orientation angle, amplitude and phase of the stripe patterns are carefully selected to combine and form a recognizable image (Figure 5c). When 65537 stripe patterns are used, means we have 257×257 stripe patterns, with the aid of Fourier transform definition, the Fourier portrait can be sketched with the resolution of 257×257 . The MR image of the Fourier portrait is shown in Figure 5d.

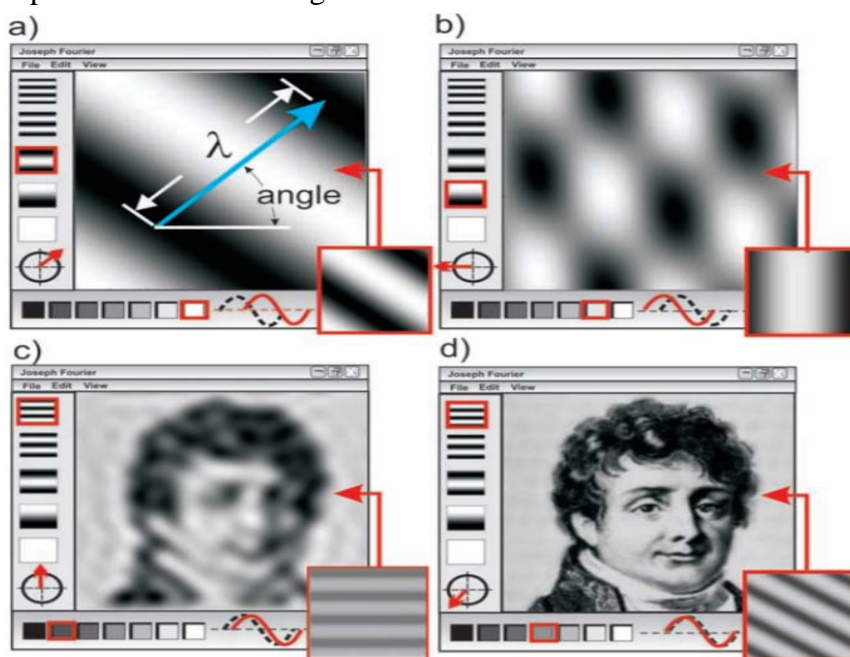


Fig. 5. The process of the MR image formation (a) a stripe pattern with specific phase, density, amplitude and orientation. (b) Second stripe is added to the first stripe in the graphical area. (c) 400 specific stripe patterns are brought in the graphical area to form the image. (d) The Fourier image with 65537, indicating the combination of 65537 stripe patterns.

As mentioned before, the combination of the various pattern stripes with specific amplitude, phase, orientation and frequency will result in the MR image. With the help of Fourier transform, the pattern of the desired image can be obtained. Each MR image can be mathematically defined as arrays related to the stripe patterns. These arrays can be written by the represented data in the "k-space" (Figure 6). As an example, a point in the k-space with the coordinates of K_x and K_y in the graphical area has a specified X and Y direction in the spatial frequency domain. Each location in the image area has a brightness which related to the amplitude of the electromagnetic wave. The points align the K_y axis denote the stripe pattern with horizontal configurations while the points with K_x alignment are representing the vertical stripe patterns. Hence, the array of spatial frequencies needed to generate the MR images is summarized in the k-space formation. In another words the k-space and the array of spatial frequencies can be transferred to each other and the MR image (Figure 7a) with its k-space format are equivalent (Figure 7b). In order to obtain the k-space data, Fourier transform is applied. And in case the MR image of the k-space representation is desired and inverse Fourier transform of k-space will result in the generation of the MR image.

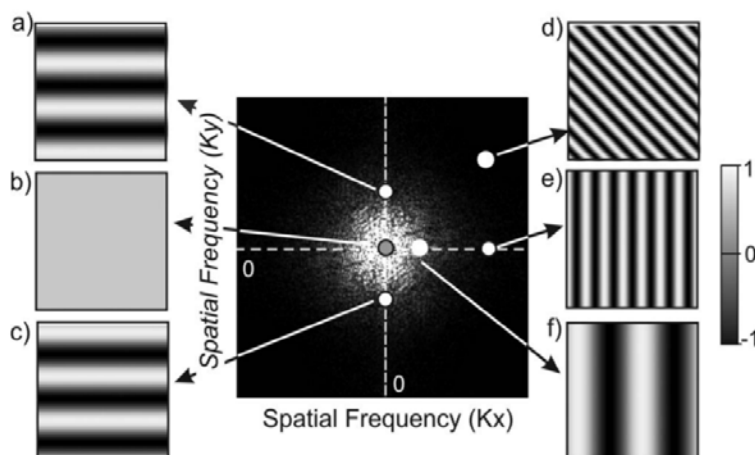


Fig. 6. Various locations in the k-space and their related stripe patterns.

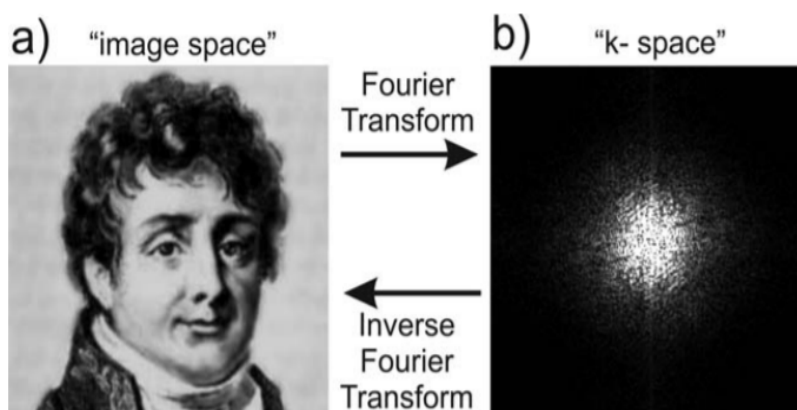


Fig.7. (a) The portrait of the Fourier and (b) its Fourier transform

Selective Excitation

As discussed earlier, the brightness of the images corresponds to the amplitude of the electromagnetic wave. In the MR images with the medical application, the three dimensional images are desired. Therefore, 3D distribution of the brightness throughout the object should be determined. This 3D brightness distribution is begin with the selective excitation. During the selective excitation, at the specific location and thickness, a section of the transverse magnetization is restricted. The process involves MR resonance, gradients and changes of the magnetic fields, and the ranges of the radio frequency (RF) excitation pulse (Figure 8). Also to obtain the shaded position illustrated in Figure 8a, the gradient of the magnetic field should be fixed. This gradient should be in the Z orientation. This will in turn result in magnetization along the Z axis with the Larmor frequency. In another word, the magnetization of the desired slice at the certain location will have a continuous range of Larmor frequencies. Their frequencies have band widths between f_1 to f_2 (Figure 8b). In the next step, an RF pulse with the band of frequencies (Figure 8c), is excite which cause the selectivemagnetization excitation with matching Larmor frequencies. Followed by this step, the magnetic field will be removed and the magnetic field returns to its original homogenous format. Here, in this situation the transverse magnetization have been generated along the specified slice and the magnetization outside the slice remains parallel to the B_0 field (Figure 8d). As a result, if the

suitable coil exist near the head, it can receive the induced signal originating from magnetization contained within the selected slice.

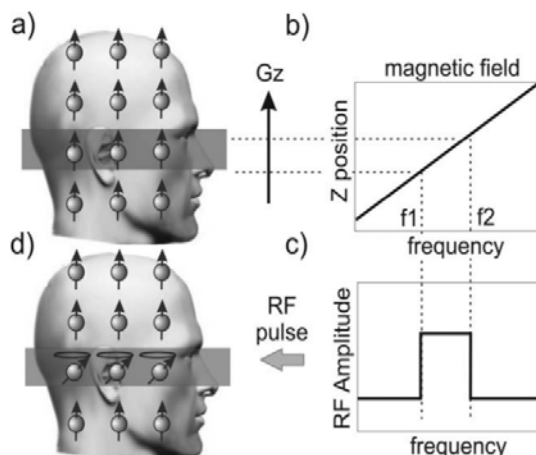


Fig. 8. The process of MR image formation in the selected region. (a) Magnetization with Larmor frequency. (b) the gradient is used to induce a position-dependent field. (c) An RF field with variable frequencies. (d) Transverse magnetization at the selected location

DIFFUSION PHYSICS

For analysis of the diffusion, first a velocity correlator is defined as below

$$\langle v(t_1)v(t_2) \rangle_c = \langle v(t_1)v(t_2) \rangle - \langle v(t_1) \rangle \langle v(t_2) \rangle \quad (1)$$

In this equation, for the sake of simplicity, it is assumed that there is no motion in the bulk. If we apply linear derivative on Eq. (1), and then use the time averaging and time translation invariant on the equation, the result will be

$$\langle v(t_2)v(t_1) \rangle = \frac{d^2}{dt_1 dt_2} \langle x(t_2)x(t_1) \rangle = \frac{d^2}{dt_2^2} \langle x(t_2)x(t_1) \rangle \quad (2)$$

The above equation in terms of the displacement can be written as

$$\langle [x(t_2) - x(t_1)]^2 \rangle = \langle x(t_2)^2 + x(t_1)^2 \rangle - 2\langle x(t_2)x(t_1) \rangle \quad (3)$$

Since the mean squared position is independent of time, we can write

$$\langle v(t)v(0) \rangle = \frac{d^2}{dt^2} [D(|t|)|t|] \quad (4)$$

This expression coincides with Equation 24 for the case of Gaussian diffusion. The definition of D, Equation 26, leads to

$$D(t) = \frac{d^2}{dt^2} [tD(t)]\theta(t) \quad (5)$$

If we solve the above equation we have

It should be mentioned that based on the Fourier transform, D(t) is the Fourier transform of D(ω). Based on this we can write

$$\langle v(t_2)v(t_1) \rangle_c = D(t_2 - t_1) + D(t_1 - t_2) \quad (7)$$

$$\ln S = - \int |q(\omega)|^2 D(\omega) \frac{d\omega}{2\pi} + \dots \quad (8)$$

$$\ln S = -\text{const} D^{\frac{1}{3}} g^{\frac{2}{3}} t(9)$$

$$S = \frac{1}{V^2} |\eta(q)|^2$$

The above equations provide a way to acquire the velocity function by assuming a delta-function for $q(\omega)$. The required parameter is the function called gradient shape $q(t)$, which is an oscillating function having the wave format, which is plotted in Figure 9. As you can see in the Figure 9, since the cosine format has two peaks, it is more preferable. As you can see in this Figure, the application of sine format is easy but it has some undesirable peak at $\omega = 0$ (Figure 9).

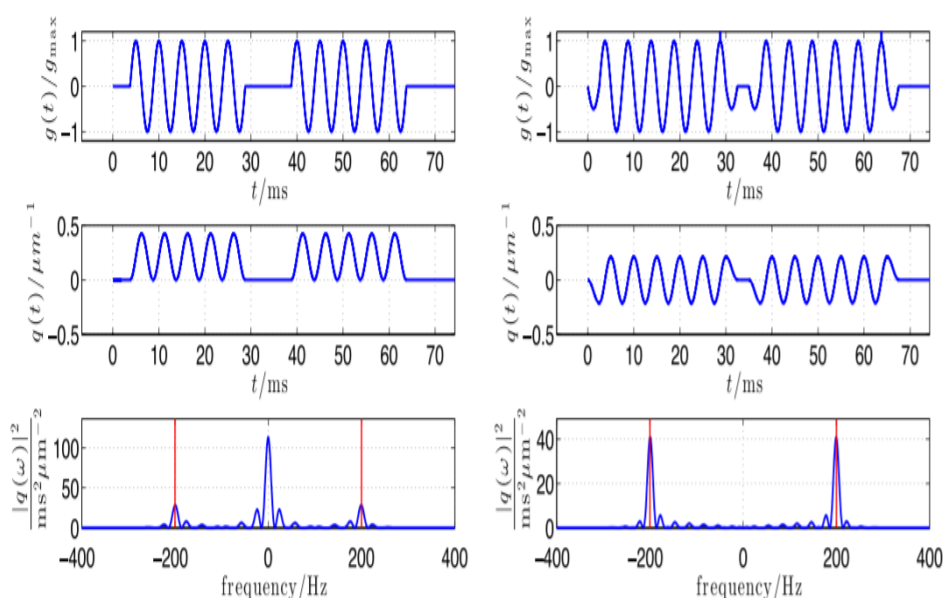


Fig. 9. Result of the simulation for MRI system with implementations of two diffusion. (Left) the sine shape which results in an unwanted peak at $\omega = 0$. (Right) the cosine format which does not induce a peak at the $\omega = 0$.

CONCLUSION

MRI has become an important tool in analyzing and probing the structure of the soft tissues and can be found in almost every hospital in advanced countries. The importance of the MRI is that it can form images of the soft tissues which the other facilities like radiology devices, CT etc. cannot analyze the soft tissue. MR images are based on the electromagnetic waves. It is important to produce and induce an electromagnetic wave and adjust the related parameters like phase, amplitude, wavelength, frequency to acquire the desired image with high resolution located at the desired point inside the tissue with the specific angle and thickness. The fundamental basis of the MR image generation is based on the Fourier transform, which transfer the basic parameter affecting the MRI to the frequency domain. With this advanced mathematical transform, the desired magnetic resonance images will become ready.

REFERENCES

- 1)Bottomley PA, Hardy CJ, Argersinger RE, Allen-Moore G. A review of ¹H nuclear magnetic resonance relaxation in pathology: are T1 and T2 diagnostic? *Med Phys* 1987;14:1–37.
- 2)S. Norozpour “Proposing New Method for Clustering and Optimizing Energy Consumption in WSN”; Published by International Journal of Talent Development & Excellence (ISSN: 1869-0459), Vol. 12, No. 3S, PP. 2631-2643, 2020.
- 3)Mehdi Darbandi; “Proposing New Intelligent System for Suggesting Better Service Providers in Cloud Computing based on Kalman Filtering”; Published by HCTL International Journal of Technology Innovations and Research, (ISSN: 2321-1814), Vol. 24, Issue 1, PP. 1-9, Mar. 2017, DOI: 10.5281/Zenodo.1034475.
- 4)Mehdi Darbandi; “Proposing New Intelligence Algorithm for Suggesting Better Services to Cloud Users based on Kalman Filtering”; Published by Journal of Computer Sciences and Applications (ISSN: 2328-7268), Vol. 5, Issue 1, 2017; PP. 11-16; DOI: 10.12691/JCSA-5-1-2; USA.
- 5)Mehdi Darbandi, A. Tarrah; “Proposing New Methods for Enhancing Cloud Security based on Evolutionary Algorithms” [In Persian]; Published by International Journal of Researches in Science and Engineering, (ISSN: 1394-0040), pp. 17-25, Mar. 2017; Iran.
- 6)Haacke EM, Mittal S, Wu Z, Neelavallib J, Chenga Y. Susceptibility-weighted imaging: technical aspects and clinical applications, Part 1. *AJNR Am J Neuroradiol* 2009;26:19–30.
- 7)Rauscher A, Sedlacik J, Barth M, Mentzel HJ, Reichenbach JR. Magnetic susceptibility-weighted MR phase imaging of the human brain. *AJNR Am J Neuroradiol* 2005;26:736–742.
- 8)Stejskal E, Tanner J. Spin diffusion measurements: spin-echoes in the presence of time dependent field gradients. *J ChemPhys* 1965;42:288–292.
- 9)Mehdi Darbandi; “Kalman Filtering for Estimation and Prediction Servers with Lower Traffic Loads for Transferring High-Level Processes in Cloud Computing”; Published by HCTL International Journal of Technology Innovations and Research, (ISSN: 2321-1814), Vol. 23, Issue 1, pp. 10-20, Feb. 2017, DOI: 10.5281/Zenodo.345288.
- 10)C. Rostamzadeh, F. Canavero, F. Kashefi “Effectiveness of Multilayer Ceramic Capacitors for Electrostatic Discharge Protection”; Published by the IN COMPLIANCE Journal (ISSN: 1948-8254), 2012; PP. 42-51; USA.
- 11)Le Bihan D, Breton E, Lallemand D, Grenier P, Cabanis E, LavalJeantet M. MR imaging of intravoxel incoherent motions: application to diffusion and perfusion in neurologic disorders. *Radiology* 1986;161:401–407.
- 12)S. Haghgoo, M. Hajiali, A. Khabir, “Prediction and Estimation of Next Demands of Cloud Users based on their Comments in CRM and Previous usages”, International IEEE Conference on Communication, Computing & Internet of Things; Feb. 2018, Chennai. DOI: 10.1109/IC3IoT.2018.8668119.
- 13)Mehdi Darbandi, M. Abedi; “involving Kalman filter technique for increasing the reliability and efficiency of cloud computing”, International WORLD COMPETITION 2012; Los Vegas, USA.
- 14)P. Shahbazi, Mehdi Darbandi; “New Novel idea for Cloud Computing: How can we use Kalman filter in security of Cloud Computing”, International IEEE Conf. AICT.; Oct. 2012, Georgia, Tbilisi. DOI: 10.1109/ICAICT.2012.6398466.

- 15) Carr H, Purcell E. Effects of diffusion on free water precession in nuclear magnetic resonance experiments. *Phys Rev* 1954;94: 630–638.
- 16) Peters RD, Hinks RS, Henkelman RM. Ex vivo tissue-type independence in proton-resonance frequency shift MR thermometry. *MagnReson Med* 1998;40:454–459.
- 17) Hahn EL. Detection of sea water motion by nuclear precession. *J Geophys Res* 1960;65:776–777
- 18) Gatehouse PD, Keegan J, Crowe LA, et al. Applications of phasecontrast flow and velocity imaging in cardiovascular MRI. *EurRadiol* 2005;15:2172–2184.